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Closed-Loop Aeroservoelastic Analysis Validation Method

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Introduction

THIS Note presents a method of closed-loop flutter aeroservoelastic analysis. The method is validated on the aircraft test model (ATM) with the STARS computer program constructed at the NASA Dryden Flight Research Center. Four main aeroservoelastic analysis software tools currently exist in the aerospace industry.[1–4] Because all of these programs were produced in the United States, a theoretical aeroservoelastic tool is also needed by the aeronautical industry in Canada. Such a tool is developed here on the basis of existent theoretical models and expertise. In this Note, we chose STARS software because it is accessible to us and because it has an interface with NASTRAN software used by the Canadian aeronautical industry. In the last few years, STARS has been applied primarily to various projects at the NASA Dryden Flight Research Center (NASA DFRC). By the use of STARS, the lateral dynamics of one-half of an ATM are studied.[1] After free vibration analysis of the finite element model of the lateral dynamics of the one-half ATM, three rigid-body modes and eight elastic modes are obtained. In the STARS software, for aeroservoelasticity studies purposes, we need to generate extra modes[1] such as three perfect rigid-body modes and two rigid-control modes. The three perfect rigid-body modes are: the rigid-body Y translation, the rigid-body roll, and the rigid-body yaw at 275 in. The two rigid-control modes are given by the aileron and rudder deflections. Aerodynamic unsteady forces of the ATM are generated with the doublet lattice method (DLM) and further converted from the frequency domain into the Laplace domain. In addition to the least-squares (LS) method, we also formulate a new conversion minimum state (MS) to LS method called the MStoLS (MS2LS) for the aerodynamic forces approximation from frequency into the Laplace domain. The MS2LS is actually a conversion of our system from state space into a transfer function form. The present formulation facilitates the integration of the obtained results with the MS method in an open loop and the validation of the MS method. This formulation guarantees that the degree of precision obtained through the MS method is not affected. On the other hand, this formulation is used only with the MS method, and by the addition of this formulation to the MS method, the computing time is greater. The results obtained through these methods are compared. Next, the results acquired with STARS of the NASA DFRC[1] are compared and validated on the same ATM with the results generated by our method programmed in MATLAB®. This comparison is good, and so our method is now validated.

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p Method: Open Loop

The p method of flutter analysis derives from the pk method of flutter analysis. The processing of iterations is the same; only the form in which the equations are presented is different. In this method, the generalized coordinates, speed and time, are normalized. The aeroelastic pk equation of motion is written as follows:

$$M\ddot{\boldsymbol{\eta}} + \left[D + (1/4k)\rho_0 c\sqrt{\sigma}V_E Q_I\right]\dot{\boldsymbol{\eta}} + \left(K + \tfrac{1}{2}\rho_0 V_E^2 Q_R\right)\boldsymbol{\eta} = 0 \quad (1)$$

where M is the mass matrix, D the damping matrix, K the stiffness matrix, k the reduced frequency, ρ_0 the sea-level density, c the reference chord, σ the air density ratio, V_E the equivalent airspeed, Q_R the real part of aerodynamic forces Q (k, Mach), and Q_I the imaginary part of aerodynamic forces Q (k, Mach). To normalize airspeeds, we introduce a variable change that will introduce reference airspeed V_0 through normal reference frequency ω_0, defined as

$$\omega_0 = V_0/c \quad (2)$$

The variable change technique converts the generalized coordinates $\boldsymbol{\eta}$ in the ω domain into $\boldsymbol{\eta}^P$ in the ω^P domain, which may be written as follows:

$$\boldsymbol{\eta}^P(\omega^P) = \boldsymbol{\eta}(\omega) \quad (3)$$

where $\omega^P = \omega/\omega_0$. Finally, by the use of the p method, we introduce the new generalized coordinates vector η^P, given in Eq. (3), into Eq. (1), and we obtain

$$M^P\ddot{\boldsymbol{\eta}}^P + \left(D^P + \nu\sqrt{\sigma}D_Q^P\right)\dot{\boldsymbol{\eta}}^P + \left(K^P + \nu^2 K_Q^P\right)\boldsymbol{\eta}^P = 0 \quad (4)$$

where $M^P = M$, $D^P = D/\omega_0$, $K^P = K/\omega_0^2$, $D_Q^P = \rho_0 c^2 Q_I$ (k, Mach)/4k, and $K_Q^P = \rho_0 c^2 Q_R$ (k, Mach)/2. In addition, ν is the airspeed ratio.

Aeroservoelastic (*p*-LS) Method in Open Loop

For the purpose of aeroservoelastic interaction studies, the idea is to convert the unsteady generalized aerodynamic force matrix Q from the reduced frequency domain $k = \omega b/V$ to the Laplace domain $s = \boldsymbol{j}\omega$. Generally, an approximation method will give new states, called aerodynamic states, that describe the dependence of the Q matrix on reduced frequency k. Various methods are available to apply these types of unsteady aerodynamic forces approximations from frequency to the Laplace domain.[1–5] We have chosen the simplest one, the LS method, because it is also applied in STARS, and, in this way, we can compare the results. First, we have to express aerodynamic forces Q in the Pade polynomials form:

$$Q(k) = A_0 + jkA_1 + (jk)^2 A_2 + \sum_{j=1}^{j=n_\beta} \frac{jk}{jk + \beta_{\mathrm{j}}} A_{2+\mathrm{j}} \tag{5}$$

where A_{j} are the coefficients of dimension equal to the Q matrix and obtained from the LS algorithm, and β_{j} are the aerodynamic lags calculated by use of various optimization algorithms. Laplace term s and reduced frequency k are given by $s = \boldsymbol{j}\omega$ and $k = \omega b/V$, and we obtain $\boldsymbol{j}k = bs/V$, which is later replaced in the flutter equation where q_d is the dynamic pressure, and we obtain

$$M\ddot{\boldsymbol{\eta}} + D\dot{\boldsymbol{\eta}} + K\boldsymbol{\eta} + q_d\left[A_0 + \frac{b}{V}sA_1 + \left(\frac{b}{V}s\right)^2 A_2 + \sum_{j=1}^{j=n_\beta} \frac{s}{s + (V/b)\beta_j} A_{2+j}\right]\boldsymbol{\eta} = 0 \tag{6}$$

Equation (6) may also be written, after normalization, in the following form:

$$\tilde{M}^P \ddot{\boldsymbol{\eta}}^P + \tilde{D}^P \dot{\boldsymbol{\eta}}^P + \tilde{K}^P \boldsymbol{\eta}^P + \nu^2 \left[\sum_{j=1}^{j=n_\beta} A^P_{2+j} X_j\right] = 0 \tag{7}$$

where the vector of aerodynamic states is defined

$$X_j = \{s/[s + (V/b)\beta_j]\}\boldsymbol{\eta} \tag{8}$$

$$\tilde{M}^P = M^P + \sigma A_2^P, \qquad \tilde{D}^P = D^P + \nu\sqrt{\sigma} A_1^P$$

$$\tilde{K}^P = K^P + \nu^2 A_0^P$$

and where $A_0^P = \rho_0 c^2 A_0/2$, $A_1^P = \rho_0 cbA_1/2$, $A_2^P = \rho_0 b^2 A_2/2, \ldots,$ $A_{\mathrm{j}}^P = \rho_0 c^2 A_{\mathrm{j}}/2$.

p and *p*-LS Methods in Closed Loop

The closed-loop configuration consists of adjoining the rigid and control modes issued from flight dynamics to the elastic modes coming from coupling between structural vibrations and aerodynamic forces. The equations for the closed-loop aeroservoelastic analysis introduce the additional term $K_g\vartheta$, which represents the rigid-body aerodynamic forces given by flight dynamics theory. This term is necessary for the control system addition to the flexible aircraft equations.

The dynamics of all modes of an aeroservoelastic system may be written as follows:

$$M\ddot{\boldsymbol{\eta}} + D\dot{\boldsymbol{\eta}} + K\boldsymbol{\eta} + K_g\vartheta + q_d Q(k, \mathrm{Mach})\boldsymbol{\eta} = 0 \tag{9}$$

where K_g is the term related to gravity and where the matrix of Euler angles is expressed as follows:

$$\dot{\vartheta} = E_v \omega_B + E_p \vartheta \tag{10}$$

where $\vartheta = [\phi \;\; \theta \;\; \psi]^T$ is the matrix of Euler perturbation angles and $\omega_B = [p \;\; q \;\; r]^T$, whereas the E_p and E_v matrices are written as functions of steady-state Euler angles and angular rates.[6]

Addition of New MS2LS Conversion

The approximation in the Laplace domain of the unsteady aerodynamic forces Q (k, Mach) in the frequency domain by the DLM with the MS method is

$$Q(k) = A_0 + jkA_1 + (jk)^2 A_2 + D[jkI - R]^{-1} Ejk \tag{11}$$

where R is the aerodynamic lags diagonal matrix, and the D and E matrices are related to the convergence of the solution. The last term on the right-hand side of Eq. (11) is further converted into the form of the last term on the right-hand side of Eq. (5), which is actually a conversion from a state-space form to a transfer function form, and may be expressed in the following form:

$$D[jkI - R]^{-1} Ejk \text{->} \sum_{m=1}^{n_\beta} B_{(m+2)} \frac{jk}{jk + \gamma_m} \tag{12}$$

This conversion is only one version of the way the MS method is computed, but we believe that it will be much easier and beneficial to implement it in this way into the closed-loop aircraft model. The lag terms β_{j} and coefficients A_{j} from Eq. (5) given by the LS method are different from the lag terms γ_m and coefficients B_m given in Eqs. (12) by the MS method. The present formulation will facilitate the integration of the obtained results with the MS method in the open loop and the validation of the MS method. This formulation guarantees that the degree of precision obtained through the MS method is not affected. On the other hand, this formulation is used only with the MS method, and, by the addition of this formulation to the MS method, the computing time is greater.

Results and Discussion

Open Loop

Table 1 shows the comparison between our results (rows 2, 4, and 5) and the results obtained in STARS (rows 1 and 2) for the following three types of analyses: 1) the aeroelastic analysis in open loop by the *pk* method (row 2), 2) the aeroservoelastic analysis in open loop by the *pk* method and the LS method of conversion of aerodynamic forces from the frequency domain into Laplace domain (row 4), and 3) the aeroservoelastic analysis in open loop by the *pk* flutter method and the MS2LS conversion (row 5). The flutter points obtained with our aeroelastic analysis in open loop by *pk* flutter method (row 2) are compared to these obtained by use of STARS (row 1). The flutter points (frequencies and damping) where flutter phenomena occurs on the ATM in open loop are given in row 2. In Table 1, results obtained on the ATM by our flutter method, modified to include the LS formulation (row 4), and called our aeroservoelastic (ASE) (*pk*&LS) method, are compared with those ones yielded by STARS software (row 3). The flutter points results obtained by the MS2LS application on the ATM model in open loop are also given, in row 5 of Table 1. When we observe equivalent airspeeds for the flutter phenomenon, in any case, the results of both of our ASE methods (*pk*&LS and *pk*&MS2LS) are closer to those of the *pk* method than those with the ASE method programmed in STARS. This is promising because the flutter points found by the application of the LS or MS2LS methods should have very close values to the ones found by the initial flutter method. Thus, we can conclude that in an open loop, we attained good comparison in terms of flutter velocities and frequencies for both flutter points.

Closed Loop

Next, we consider the ASE system of the ATM in closed-loop configuration, after the application of the LS method to the p method. We first choose to use the LS formulation in the p method

Table 1 Aeroelastic and aeroservoelastic analysis

	Flutter point 1		Flutter point 2	
Method	Equivalent airspeed, kn	Frequency, rad/s	Equivalent airspeed, kn	Frequency, rad/s
STARS–*pk*	441.7	77.4	863.4	147.1
Our *pk*	441	77.4	827.2	142.8
STARS–ASE	474.1	77.3	728.9	136.2
ASE (*pk*&LS)	441.9	77.5	885.2	138.5
ASE (*pk*&MS2LS)	444.4	77.5	907.2	135.8

in a closed loop (our ASE method). We get flight conditions for ASE analysis in a closed loop that are identical to those used by STARS.[1]

The flutter speeds obtained with our three methods, namely, 1) the flutter p method, 2) the LS forces approximation applied on the flutter p method (our ASE p&LS method), and 3) the MS2LS unsteady forces approximation applied on the flutter p method, were found to be very close to each other (values of around 260 kn). The slight difference is explained by the precision of LS or MS2LS methods applied to the ATM.

Conclusions

Four types of analyses were conducted on the ATM to characterize flutter in the ASE context. ASE analysis by the pk method first introduced the ATM and verified the pertinence of its data and the results obtained by STARS software at the NASA DFRC.[1] In fact, a comparative study of the values of aeroelastic analysis applied on the ATM by the pk method and the aeroelastic results with STARS revealed good coherence at the level of flutter prediction in an open loop. After this analysis, the ATM was found to be consistent.

ASE analysis on the ATM by the p method in a closed loop allowed the validation of its ability to represent complex ASE systems and to predict the flutter phenomenon in closed-loop ASE systems. Comparative study of the results of ASE analysis applied on the ATM by the p method, and of the ASE values in STARS, revealed a prediction relatively identical to a flutter phenomenon in a closed loop very close to 260 kn. This analysis shows the importance of taking the chain of control, the dynamics of actuators, and the sensors in the flutter prediction phenomenon into account, as suggested by the p method in a closed loop, because the flutter phenomenon appears at a much weaker speed in a closed loop (263 kn) than it does in an open loop (441 kn). Finally, the two last analyses allowed us to test the impact of aerodynamic forces approximated by the LS or MS2LS methods on flutter phenomenon prediction, first in an aeroelastic context, and later on in an ASE context. Also, aerodynamic forces approximation used in the analyses, which consists of LS minimization, is not optimal. The precision of aerodynamic forces approximation may be increased significantly by augmention of the number of aerodynamic lags on the one hand, or by the use of the most powerful approximation methods that guarantee good precision by decreasing the number of aerodynamic modes on the other. Detailed validation of an ASE analysis method is presented in this Note as is a new conversion of the MS into the LS form called MS2LS.

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Thick Wings in Steady and Unsteady Flows

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Introduction

SOLUTIONS of full Navier–Stokes equations for three-dimensional bodies in unsteady flowfields are challenging. One method that is available to compute the unsteady flowfields for arbitrary bodies is the unsteady vortex-lattice method (UVLM). When the pressure distribution on a thick wing is not required and the lift and moment coefficients are sufficient, often a thick wing is replaced by a lifting surface. By this approximation, one can realize substantial reductions in computational time. The question is when such an approximation appropriate and how thick of a wing can be replaced by a lifting surface? In this Note, the effect of thickness on the aerodynamic lift and moment is investigated. The general, unsteady, three-dimensional, vortex-lattice method is evaluated by making a number of comparisons between numerical results obtained from UVLM and experimental data. The pressure coefficient of a thick wing using UVLM is compared to the exact solution. Both steady and unsteady flows are considered, and both thick-wing and lifting-surface versions of UVLM are used.

Analysis

When a wing of arbitrary planform is set in motion, vorticity is generated in a thin region adjoining the surface known as the boundary layer. Some of this vorticity is shed into the flow and becomes the wake. It follows from the definition of vorticity that vorticity anywhere in the flowfield induces velocity everywhere in the flowfield. This is a kinematical result and valid for viscous as well as inviscid flows. In the actual flow, the vorticity-induced disturbance emanating from the boundary layers and wake interfere with the oncoming stream to the extent that both the no-slip and no-penetration boundary conditions are satisfied on the surface of the wing.

In the present aerodynamic model, vortex sheets wrapped over the body surfaces simulate the boundary layers. These infinitesimally thin layers of vorticity may be viewed as the infinite Reynolds number approximation of the actual boundary layer. Hence, one can expect the present model to improve as the Reynolds number increases. To expedite the calculation of the induced-velocity field, the sheets are replaced by lattices of discrete vortex lines. Each line segment in the lattice is relatively short and straight, and its induced velocity is readily calculated from the familiar Biot–Savart law.

The circulations around the individual segments of the lattice are determined by imposing simultaneously the no-penetration condition at the centroid of the corners in each element and spatial conservation of circulation. For the pressures on the upper and lower surfaces of the body along the sharp trailing edges to be the same, vorticity must be shed. This shed vorticity is convected away from the wing and constitutes the wake. The flow is irrotational everywhere except in the boundary layer of the body, where vorticity is being generated, and in the wake. The flow in the wake is considered to be inviscid and rotational; hence, to make the wake force free, the vorticity is convected with the fluid particles. The present method

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